

Genetic Algorithms and Financial Crises in Emerging Markets *

Thierry APOTEKER and Sylvain BARTHELEMY †

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Abstract

Ratings using a linear or non-linear combination of selected economic indicators (real exchange rates, reserves over M2, imports, short-term debt level...) can be very useful to understand and quantify country risk levels, but are not sufficient. Sudden changes in indicators or combinations of indicators levels can induce a higher risk than what a simple linear model could measure and non-linear models using thresholds, like SETAR or EXPAR, are difficult to use with poor data and require a lot of observations. That's why we decided to create a tool that would help us to make a better diagnosis on country risk without an equation or a thresholds model. We called this indicator the Vulnerability, which is measured as a distance to an optimal combination computed as a risky pattern by a genetic algorithm regularly updated with new official data.

I INTRODUCTION

In the 1970-1990 period, country risk was mainly a matter of sovereign risk. Evaluation of this sovereign risk was done using a combination of two macroeconomic variables : a measure of the total external debt and a measure of the current account deficit (analysts usually used total external debt over exports for the first indicator and current account over exports for the second one).

The capital flows liberalisation of the 1990s, radically changed the traditional view of sovereign risk. As shown on figure 1, private capital flows have become more and more important since 1989-90, and borrowers now have an "unlimited" access to international financial resources. Because of the new financial landscape, and as country risk is not only an economic *game* between lenders and the public sector, analysis must include more complex economic indicators. Country risk is a combination of macroeconomic and microeconomic factors and not only an external financial equilibrium. In this perspective, to

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†Thierry APOTEKER (apo@tac-financial.com) is the Managing Director of T-A-C and Sylvain BARTHELEMY (barth@tac-financial.com) is a T-A-C Economist. They can be reached at *La Saigeais, 35140 Saint Hilaire des Landes - France*, or <http://www.tac-financial.com>

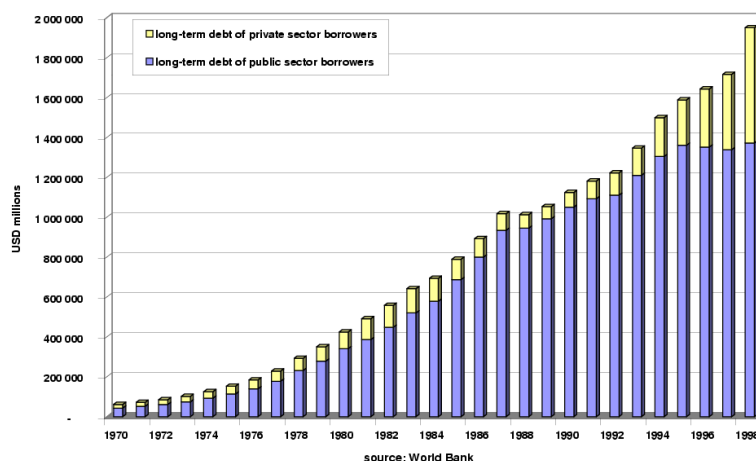


Figure 1: External debt for all developing countries, 1970-98

create RATINGS and good early warning indicators you must take into account many more parameters than before, without taking all these indicators together to avoid “smooth” warnings, which perform very poorly.

A lot of theoretical and empirical work has been done to explain the financial crises of the 1990s and to search for early warnings that could help to make good predictions of financial crises in emerging markets (balance of payment crises, exchange rates crises...) : Eichengreen et al. (1995), Kaminsky & Reinhart (1996), Frankel & Rose (1996), Goldstein (1996), Kaminsky et al. (1997). Some important consensual macroeconomic indicators may help to understand the route to crisis. Real exchange rate overvaluation, for example, is one of the most consensual variables, so are reserves, M2 (money plus quasi money), exports, imports... One of the major problems encountered on using these indicators is that none of them is permanently a good indicator.

Real exchange rate overvaluation is a necessary condition but not sufficient for a currency crisis. If, for example, this overvaluation is observed with a simultaneous combination of a high current account deficit and a decrease in forex reserves, probabilities for a currency crisis are high. Conversely, exchange rate overvaluation can be sustained if reserves are growing because of external surpluses. It is interesting to observe that most of the recent academic literature has focused on crisis indicators combining exchange rate levels and changes in reserves, even though from an operational point of view, a crisis is very different if it is limited to a drain on foreign exchange reserves, or if it indeed includes a major devaluation.

Overall, both recent academic research and empirical observation have highlighted the non-linear characteristics of the relationship between macroeconomic variables and the degree of country risk. In a given set of economic circumstances, even a small change in a key indicator, if moving over a significant

threshold, can trigger a large crisis.

The combinatory approach (based on the "if...then" type of relation) implies however that the number of variables is limited, otherwise the number of possible combinations is incredibly high. The basis of the T-A-C approach was therefore, to concentrate first on building a small number of very powerful indicators and exclude variables which exhibit strong correlation between them, and second to establish the proper relationship between these indicators including thresholds and non-linear effects. We had developed an initial country risk assessment service using a formal non-linear tool in 1994-95, which has been effectively used with some of our long-lasting customers since 1996. The "true test" of the method came with the Asian turmoil in 1997 : we very correctly predicted the Thai crisis as early as February 1997 (when the consensus was still quite positive on the country's ability to muddle through), as well as the Korean banking risks in June 1997. However, we missed the magnitude of the crisis and its spreading so rapidly to other Asian countries.

This is why we decided in November 1997 to initiate a new round of fundamental research into country risk analysis, which lasted until the end of 1999. This 2-year research has given birth to the present RISKMONITOR service.

The objectives and constraints set for the research were :

- To improve the predictive capacity of risk RATINGS
- To provide a simple and understandable framework of analysis for our customers
- To fully explore all recent development in non-linear signaling theory and practice
- To use only published and internationally recognized statistics
- To use the same methods and criteria for the whole universe of 40 major developing countries

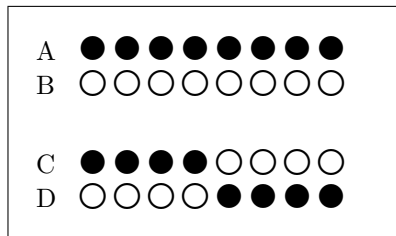
The following sections describe the basic foundation of our VULNERABILITY measure, computed with genetic algorithms.

II GENETIC ALGORITHMS

The first simulations of genetic systems on computers began in the 1960s, however, genetic algorithms (GA) were first introduced by Holland (1975). The genetic algorithm is a machine learning optimization method based on a metaphor of the evolution process observed in nature. These search procedures are based upon the evolution of a population of individuals (a vector of possible solutions). GA are part of evolutionary supervised learning methods, as they require an evaluation function provided by the creator of the algorithm (instead of reinforcement learning agents for example).

First, you create a population of individuals where each individual is represented by a character string. These strings represent the *chromosomes* or

Figure 2: Single Point Crossover : parents are (A,B) and childs (C,D)



genotypes and each character represents a *gene*. The algorithm simulates a process of evolution with this population. The optimization process is achieved through the minimization or maximization of an evaluation function which permits measuring the performance of each individual (or string) and gives a fitness value.

- Initialize a *population* of *individuals*.
- Evaluate each *individual* of this *population*.
- Select best *individuals* and do random changes.
- Test for stopping criterion : return the *solutions/individuals* if satisfied or goto step 2 if not.

Selection permits to select the basic individuals of the next generation. The most common method, initiated by Holland (1975), is the *Roulette Wheel* selection which is a fitness-proportionate selection method : the number of times an individual will be selected is equal to the fitness divided by the fitness mean of the whole population. Many new selection algorithms, much more advanced, are now available : *sigma scaling*, *Boltzmann selection*, *rank selection*, *tournament selection*... We won't explain all these methods as we decided to use the *Roulette Wheel* selection which is a simple and efficient selection method for combinatorial analysis.

Crossover represents reproduction in a genetic algorithm. After the selection of individuals, some individuals are chosen to cross parts of themselves. It is the major instrument of innovation in the genetic algorithm. It will try to prevent the population from moving toward a local optimum. The most common way to do crossover is the *single point crossover*, illustrated in figure II. The crossover rate is usually high, about 70% to 95% of the population.

As we better understand how GA work, mutation is taking a better place. Mutation won't replace crossover but is a good complement and the scientist must take time to find a good mix between crossover and mutation. This parameter is usually low, between 0.5% and 1% of the whole population.

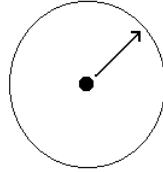


Figure 3: Simulated Annealing

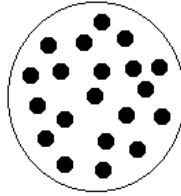


Figure 4: Genetic Algorithms

Genetic Algorithms are not widely used by economists who usually prefer using Neural Networks for financial applications, but some examples can be found on Neely et al. (1996), Pictet et al. (1996) and Wallet et al. (1996). When you have to solve a combinatorial problem, and particularly an NP-Complete, problem which is not solvable in a polynomial time, GA will be useful. A very well known example of problem which is always presented as the combinatorial problem that GA can solve faster than standard algorithms is the *Traveling Salesman Problem*, where a man wants to find the best combination of towns to visit, minimizing the distance by choosing the right ordering.

GA are not designed to create models, like standard Neural Networks or Genetic Programs but to get *topologies* or *selections*. Another well known *artificial intelligence* method used to search for an optimal combination is the simulated annealing, which is quite the same but it uses a different way of moving around to find an optimum. Simulated annealing is a neural network search procedure using only one individual, moving around a possible solution with random increments, as shown on figure 3, and choosing the best evaluation as the next position. Genetic algorithms search for an optimum with a vector of individuals (a vector of possible solutions), and the vector will *follow* the best evaluations (taking care of local optima), as shown on figure 4.

III DATA AND METHODOLOGY

We use RISKMONITOR indicators which provide quantitative as well as a graphic comparison of 40 major developing countries based on T-A-C proprietary non-linear methodology.

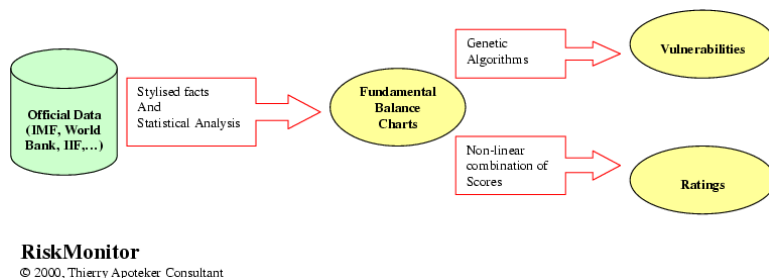


Figure 5: RiskMonitor Scheme

Figure 5 shows the basic scheme of RISKMONITOR. We start only with official data and we use five official data sources : International Monetary Fund, World Bank, OECD, United Nations Statistics and Bank for International Settlements. All these data are put together into a very large economic database system called RED (*Risk-Economic-Database*).

It is important to stress that, as a starting point, we want to use data which are comparable from one country to another, which implies that we do not rely on national sources, but only on these recognized international institutions. We also recognized at the initial stages of the research that many mistakes are made when putting estimates and forecasts in a complex model. Most often, the predictions made are the reflection of the intuitive perception of the future risk by the analyst¹ ; revisions and “surprises” have been so frequent that we considered it would be much more efficient to devise indicators and quantitative measures that incorporate a predictive value. Considering this, it is essential to note that all our results are computed by fully taking into account the existing time gap between the actual availability of economic indicators and the date to which they refer (i.e., if GDP growth is used in one of our indicators, and the data is available only with a one year lag, we measure the signal at a given date by using GDP growth figure for the preceding year). The predictive power of the service is, therefore, not limited or affected by the uncertain quality of the forecast.

Theoretical and empirical analyses led us to retain five major FUNDAMENTAL BALANCE CHARTS which represent the basis of the RISKMONITOR methodology, and which are all based on a combination of economic indicators constructed from the mentioned international sources.

- G1 - the Growth Balance,
- G2 - the Financing Balance,
- G3 - the Foreign Exchange Balance,
- G4 - the Cyclical Balance,
- G5 - the Banking System Balance.

¹Convincing examples are Russia and Brazil before and after they were hit by a crisis.

Each of the five charts is built as a scatter graph based on two different composite indicators illustrating an easy-to-understand economic feature of the country, with a "risk threshold" defined for each of them enabling to represent non-linearity. Both the indicators and thresholds were incrementally adjusted to obtain the best statistical fit with existing crises.

The Growth Balance measures the ability of a country to record adequate economic growth without involving excessive imbalance in the external financing.

It combines a domestic growth indicator (GDP growth per capita, with an accelerating effect) with an external balance indicator (current account balance, as a percentage of current account receipts, excluding the imported share of exports for goods).

The Financing Balance measures the quality of the external financing of a country and the risks associated to external debt.

It combines a financing stability indicator (putting the respective share of foreign direct investment and short-term debt in relation with foreign exchange "surpluses") with two indicators of debt service: the first one carries information of a rather "structural" type and relates to standard debt service (total interests on external debt and annual depreciation of medium- and long-term loans as an average percentage of GDP and of current account receipts excluding the imported share of exports for goods; this indicator appears in dotted line and articulates around a threshold of 15%). The second indicator of debt-service relates more to the short-term aspect of the external financing risk : the maximum potential service adds to the standard debt service the assumption that all short-term loans will not be renewed during the year and will thus have to be paid back to the lenders (the maximum potential service is a percentage of current account receipts excluding the imported share of exports for goods, with a risk threshold of 60%).

The Foreign Exchange Balance enables a more accurate and short-term measurement of the financial component of country risk. In fact, financial difficulties, all the more so crises, are always preceded by significant imbalances on the foreign exchange market.

These imbalances are measured from the combination of an indicator of foreign exchange reserves (evolution of official foreign exchange reserves in relation with the domestic money supply and international interbank lending) and of an indicator of exchange rate pressure (a complex indicator of competitive pressure on the country's exchange rate, taking into account the currency moves of the main countries also active on major economies, in addition to a "reference" exchange rate dollar against euro or yen, depending on the country). Considering the sensitivity of country risk to the Foreign Exchange Balance, "secondary"

thresholds are used to graduate the non-linearity of the measure.

The Cyclical Balance provides a measurement of the cyclical position of the economy in terms of country risk analysis, and enables the measurement of the quality of the domestic economic policy and of the nature of the most sensitive risks.

It combines a monetary pressure indicator (domestic liquidity in relation with the previous inflation and economic growth rates) with an indicator of real economic pressure (a leading indicator of the pressure in domestic demand, built from moving elasticities to imports and the exchange rate).

The Banking System Balance (G5) enables to measure the risks associated with the global imbalances in the banking sectors in terms of systemic risk.

It combines an over-indebtedness indicator (share of bank lending in the overall economy) with an indicator of foreign financing (the dependence of local banks on interbank refinancing through international borrowings).

Using these five FUNDAMENTAL BALANCES we measure country risk using the RATINGS on the Development risk, the Solvency risk, the Short-Term Financial Risk and the Short-Term Cyclical risk. They use non-linear formulas to incorporate thresholds movements informations. These RATINGS are based on an expert reasoning over empirical risky situations but not on an exhaustive search over the whole possible situations. We decided to search for a second warning indicator which would measure the number of satisfied conditions over an optimal risky combination : what we call the VULNERABILITY.

IV IDENTIFYING VULNERABILITIES

We have identified four types of crises : transfer crises, liquidity crises, exchange rate crises and cyclical development crises.

- **Transfer crises**, are London Club rescheduling.
- **Liquidity crises**, are characterized by unvoluntary *new money*.
- **Exchange rate crises**, occur with large drops in real exchange rate: if the real exchange rate move over the quarter exceeds 20%, or 30% over two quarters, or 40% between three and six quarters.
- **Cyclical development crises**, occur with a sharp decline in GDP growth: when current GDP growth is under 1/3 of the previous year growth rate and if current GDP growth is 3.5% less than the previous year growth rate and if current GDP growth is under 5%.

We search for patterns using combinations of the four basic quads of each graph². If x represents the first indicator, y the second, a the x threshold and b the y threshold, the quadrics for a graph are :

- quad 1 if $x < a$ and $y \geq b$;
- quad 2 if $x \geq a$ and $y \geq b$;
- quad 3 if $x \geq a$ and $y < b$;
- quad 4 if $x < a$ and $y < b$;

As you add years or graphs, the number of combinations grows as a power function, so this problem is well suited for an optimisation with a genetic algorithm because it's a matter of selection into a very large space of possible solutions.

Range of possible states for characters in the strings (*allele values*) are always booleans and are 0 if we don't take the quad into account or 1 if we always take the quad into account. For example, the string 1110 denotes that you must take into account positions over quads 1, 2 and 3 and not 4. We decided to search for patterns over 2 years : the year in which the crisis occurs and the previous year. So, as there are 3 annual graphs and 3 quarterly graphs the number of boolean values in a string is 120 and so this give a total of 2^{120} possible combinations³.

We did the calculus using an objective function. The purpose is to make a compromise between the number of included quads and the power of the evaluated combination. We simply use a linear combination of these two measures as the evaluation function as it allows to change easilly preferences parameters.

$$E_c = \beta B_c + (1 - \beta)H_c \tag{1}$$

where E is the evaluation of the c combination, B is the bonus for this number of quadrics and H the percentage of good hits in this combination.

The percentage of good hits H is evaluated using the crises presented at the begining of this section (transfer crises, liquidity crises, exchange rate crises and cyclical development crises). Each individual of the population of the genetic algorithm is evaluated with this objective function. If there is a crisis detected for one of the 40 countries tested at a given period, the individual evaluated will test the number of hits into the combination it represents (for the current year and the year preceding the crisis) over the total number of hits (excluding non-available data) ; the resulting number will be H .

The bonus B is just here to increase the power of the combinations using a small number of quads. The table 1 shows the bonus level for each number

²There are 2^4 possible combinations of *lighted* quads per graph, because we can use more than one quad per graph.

³There are 4 characters per period (because there are 4 quads) and per graph. As we have 3 annual graphs and 2 years we have $(4 * 3 * 2) = 24$ characters for annual graphs. We have 3 quarterly graphs, and height quarters so $(4 * 3 * 8) = 96$ characters for quarterly graphs and so a total of $24 + 96 = 120$ characters

of quads. Bonus 1 if the combination uses 1 or 2 quads per graph and 0 more or less, indicates that the individuals of the algorithm will always prefer using 1 or 2 quads per graph because evaluation will be higher. The β parameter is here to quantify the preferences of the individuals between the bonus and the percentage of good hits.

Number of quads per graph	Bonus (B)	Percentage of good hits (H)	Total (E)
0	0	0	0
1	100	h	$100\beta + (1 - \beta)h$
2	100	h	$100\beta + (1 - \beta)h$
3	0	h	$(1 - \beta)h$
4	0	h	$(1 - \beta)h$

Table 1: Evaluation Criterion

To do this, T-A-C have created a proprietary genetic algorithm system to search for the risky combinations : GASV, *Genetic Algorithm Searching for Vulnerabilites*. We used a cluster of computers -running MPI- to run the algorithm (the system sometimes runs more than a week to find an optimal combination). Parameters of the algorithm changed slightly between the types of crises tested but were :

- A population size between 50-100 individuals. Unsuccessful tests have been done with more than 100 individuals.
- We have used a simple *roulette wheel* selection mecanism.
- The crossover probabilities tested were 0.7, 0.8 and 0.9 and we obtained the best results with 0.7.
- The mutation rate chosen were 0.005.
- We tried many different β values (from 0.1 to 0.9) and we obtained good results with the medium value 0.5.

The results for annual graphs are presented on table 2. Sometimes genetic algorithms find the optimum fast and sometimes they are slow to converge. That's why there is a column that shows the *speed of convergence*. The low speed of exchange rate crises indicates that it's more difficult to find a good exchange rate crises combination than for cyclical crises, liquidity crises or transfer crises.

The table 2 shows the quads that are usefull to monitor when you want to detect VULNERABILITIES on one of the four types of crises tested. For the liquidity crises for example, when indicators for G1, G2a or G2b are on quad 3 (bottom right quad) for the current year and on quad 1 or 3 (top left or bottom right) for the previous year on G2a and quad 3 (bottom right) for the previous year on G2b, then the VULNERABILITY for liquidity crises is high.

	G1 t,t-1	G2a t,t-1	G2b t,t-1	speed of convergence
exchange rate crises	0000,0000	0010,0000	0010,0010	low
cyclical crises	0001,0000	0010,0010	0010,0010	high
liquidity crises	0010,0000	0010,1010	0010,0010	high
transfer crises	0000,0000	0010,0010	0010,0010	high

Table 2: Annual Results

All these combinations are updated regularly and used as a complementary indicator of our RATINGS. The next section shows some of the results obtained using these VULNERABILITIES on some developing countries.

V RESULTS AND APPLICATION TO SOME DEVELOPING COUNTRIES

VULNERABILITIES computed with these optimal combinations are a complementary tool to the RATINGS. As shown on figures 6, 7, 8 and 9, we compute an indicator of VULNERABILITY. This indicator express a percentage of fulfilled conditions of all the requirements found by the genetic algorithm. All these percentages are computed for observation dates, using only official data available at the given period and without estimates.

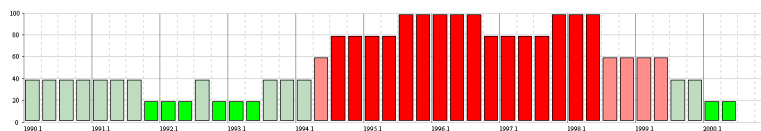


Figure 6: Mexico, % of fulfilled conditions / transfer crisis

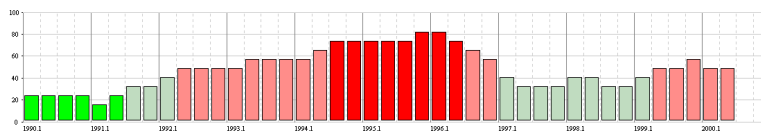


Figure 7: Mexico, % of fulfilled conditions / liquidity crisis

VULNERABILITIES are combinations, so they should be expressed as dummy variables, but we use a distance from the current combination to the optimal combination, because it's a simple indicator of *how far is the VULNERABILITY* from the current state.

As shown on the figure 9, the cyclical crisis VULNERABILITY for Mexico was very high since the second quarter of 1994, and liquidity and transfer crises were

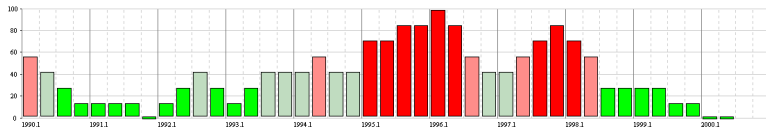


Figure 8: Mexico, % of fulfilled conditions / exchange rate crisis

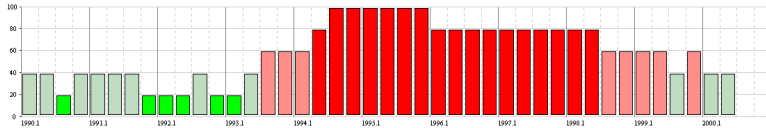


Figure 9: Mexico, % of fulfilled conditions / cyclical crisis

high too, but the exchange rate crisis VULNERABILITY is not as high as the two others.

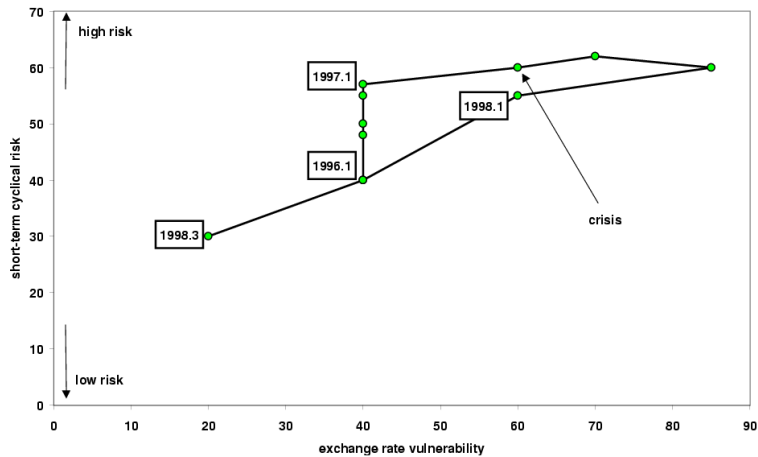


Figure 10: Thailand, Ratings and Vulnerabilities before the 1997 crisis

The figure 10 shows a scatter graph of the RATINGS and VULNERABILITIES for Thailand before the 1997 currency crisis. The VULNERABILITY is high but static from 1996.1 to 1997.1 but the risk is higher and higher⁴. Then, the risk remains stable and the VULNERABILITY becomes very high too : from less than 40% to more than 60% of the total number of conditions are fulfilled in 1997.2 and 85% in 1997.4. After this quarter, the RATING and the VULNERABILITY decrease to 30 and then 20 in 1998.3.

⁴The risk RATINGS are measured from 0 which is the lowest risk to 100 for the highest risk. As these risk RATINGS are non-linear combinations of SCORES for FUNDAMENTAL BALANCES, a risk over 45 is a very high risk and countries have rarely a RATING over 60.

The last sequence with Thailand, that cross RATINGS and VULNERABILITIES, shows that these two measures are to be taken by pairs. The RATINGS are always necessary to quantify risks and the VULNERABILITIES can add a signal which act as a *risk reinforcer* but never as a stability signal.

VI CONCLUSION

This paper shows that using genetic algorithms to find patterns or combinations for a country risk early warning system can be helpfull and somewhat easier than a common statistical analysis.

The vulnerabilities assess the unexpected side of country risk sudden changes. If ratings are created taking economic indicators as a basis of the analysis, and are verified with crises, the vulnerabilities are computed starting from crises to find risky combinations of economic indicators. The vulnerabilities are not a stability indicator when they are low but only a *risk reinforcer* if they are high, a kind of supplementary warning.

RiskMonitor vulnerabilities were computed on large quads and it may be interesting to compute more precise combinatorial paths, without trying to create new fundamental equilibrium scores.

We have also identified two other types of crises that would be interesting to add to these vulnerabilities : Paris Club Rescheduling and Banking Crises.

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